Introduction

The purpose of this project is to identify and measure risks associated with the Guaranteed Minimum Accumulation Benefit on a Deferred Variable Annuity. The project proposes a risk management strategy to reduce the GMAB risk that an insurance company bears. In order to meet this objective, this paper describes the process of the risk management, which includes the steps of identifying possible risk factors, quantifying the risk exposure, electing the most cost efficient risk management strategy among alternatives and evaluating the risk management system.

A small model was built to demonstrate how to quantify the risk by the means of Value at Risk (VaR) and CTE, and to assess the proposed risk management strategy. The pre-packaged economic scenarios, provided by the American Academy of Actuaries for C-3 Phase II, were used. The Model also incorporates the RBC C-3 Phase II capital requirement. Policyholder behaviors are assumed to be rational to a certain extent. For easy reference, this model is referred as “The Model” throughout this paper.

Utilizing the Monte Carlo simulation method, The Model is able to quantify the market risk in terms of VaR and CTE. Pricing risk is also assessed against the fair market value of this embedded option. The Model measures the profitability of the GMAB rider by IRR and Embedded Value. Several stress tests were performed to measure the risk exposure under various assumptions, and to catch the characteristics of a real option that is embedded in the GMAB. The results of The Model are also used to examine the proposed risk management strategy.

Risk Management Process

1. Identify Risk Factors

This project is based on a Guaranteed Minimum Accumulation Benefit rider (“the Rider”) offered on deferred variable annuities. The Rider guarantees that, if the policyholder surrenders their policy on or after the tenth policy anniversary after the rider is elected, the cash surrender value will not be less than a specified guaranteed amount - even if the current account value is below the guaranteed amount. The guaranteed amount is the account value at the time the rider is elected, reduced proportionally for any withdrawals after the rider is elected.

When the equity market goes down, the policyholder’s account value might be significantly below the guaranteed amount. The insurance company would then experience large claims and excessive lapses in the tenth year. The domestic equity market risk is substantial. However, the other financial risks cannot be ignored either, since the policyholder can invest in several separate account choices, which include the domestic equity, bond, money market, international and specialty funds, offered by the insurance company. In addition, a lower interest rate, typically a long duration risk free rate, increases the embedded option value considerably. If the policyholders allocate more money to the separate account, the volatility of the policyholder account value will increase. It also forces the insurance company to sell assets in the general account portfolio, usually at a loss. Thus, in many deferred variable annuity contracts, insurance companies may limit the ability of policyholders to transfer money from the general account to the separate accounts.

The risk from the policyholders' behavior is hard to hedge with a financial instrument. The policyholders can transfer their account value among several funds. That may cause some difficulties in finding right hedging vehicles. The contract design gives the policyholder the option to cancel or to extend the rider. If the GMAB guarantee were in-the-money, rational policyholders would not surrender their policy during the first ten years. They may surrender their policy in the tenth year if the amount in-the-money is significant. The lapse rate may increase when the guarantee is deep out-of-the-money, since the policyholders would not continue to pay the rider charge for a guarantee that has no value.

2. <u>Quantify Risk Exposures</u>

Many risk quantification tools have been used to measure risk exposures. The common tools used in the insurance industry include Value at Risk (VaR), Duration, Credit Rating, Liquidity Analysis, Tail VaR, Conditional Tail Expectation, Economic Capital, Scenario Analysis and Stress Testing. Sharp Ratio and drawdown are also common tools used in some financial institutions. I will implement VaR methodology, since it is a powerful approach and has far-reaching uses. I complement VaR by CTE as another risk measure. To measure profitability, I also include the IRR and Embedded Value measure.

VaR states how likely it is that the loss exceeds VaR amount over a period of time. It provides a common, consistent, and integrated measure of risk across risk factors, and asset classes, leading to greater risk transparency and a consistent treatment of risks across the firm. It also provides an aggregate measure of risk. Unsurprisingly, VaR has its own limitation as a risk measure. VaR doesn't give an indication of the magnitude of the potential losses in the tail. When the loss distribution is bimodal, VaR may be very small at a low confidence level, but jump to a very high value for a higher confidence level. Therefore, I also include CTE to make up for the drawback of VAR measure. The CTE (y) is calculated by averaging the loss amounts greater than y percentile loss amount; e.g., CTE 65 is the average of the 35% greatest losses.

There are three common approaches to calculate the VaR and derive the loss distribution: 1) the Delta-Normal approach, 2) the historical simulation approach, and 3) the Monte Carlo approach. In addition, Back-testing the VaR model and Scenario Analysis are valuable tools of risk-management systems, along with the VaR measurement.

The Delta-Normal approach assumes the return of a portfolio is normally distributed. It involves estimating the covariance matrix of the risk factors. The advantages of Delta-Normal approach are that it is computationally attractive and easy to implement. However, it might underestimate the true value at risk due to the fat tails of returns for many financial instruments. It is not able to capture the asymmetric distribution of returns for nonlinear instruments such as options. Since the GMAB guarantee is an embedded option, Delta-Normal is not a proper approach.

The historical simulation approach uses the actual historical data to construct the hypothetical future returns. The expected change of the current portfolio is calculated based on the full set of hypothetical future returns. The VaR is then generated from the entire distribution of the portfolio values. If the historical data is available, this historical simulation approach is simple to apply. This approach allows nonlinearities and non-normal distributions of returns. The fat tails in the distribution also are taken into account. With sufficient historical data, it can also deal directly with the choice of horizon for measuring the VaR, instead of assuming that the return follows the Brownian motion. The requirement for sufficient historical data is a major drawback of this approach. Moreover, if the sample size is too small, the estimation error can be significant since the VaR is a statistical estimate. In additional, if the time window of taking samples omits important events, the tail risks will not be well represented. This approach also assumes that the past represents the immediate future fairly, which is unrealistic. Due to lack of sufficient historical data for the Rider, the historical simulation method cannot be used for this analysis of risk of the Rider.

With significant improvements in computer power, the Monte Carlo simulation approach has become the most powerful method to calculate VaR. It is similar to the historical simulation, but it does not require the historical data of the asset returns. Instead, it creates a hypothetical return of assets based on a random draw from a pre-specified stochastic process. The flexibility of the Monte Carlo method allows us to incorporate the fat tails and the extreme scenarios. Furthermore, it can also integrate a wide range of exposures and risks. The computational time is the major drawback to implementing the Monte Carlo simulation; however, this problem becomes less and less severe. The Monte Carlo approach assumes that the underlying risk factors follow a specific stochastic process. Modeling errors potentially exist for risk factors that may not follow the assumed processes in the real world. The pre-packaged economic scenarios, provided by the American Academy of Actuaries, were used here to perform the Monte Carlo analysis on the GMAB risk embedded in the Rider.

The VaR from the approaches mentioned above is basically an estimate. To assess how accurate the VaR is, the most appropriate method is VaR Back-testing. The VaR Back-testing procedure involves comparing the actual losses for a given time period observed by the insurance company with the calculated VaR. Since the Rider guarantee becomes effective ten years after this rider is elected, and we have only been issuing this rider for 7 years, the historical data is not enough to perform the Back-testing. The VaR quantifies the possible loss under the "normal" market conditions. It may fail to catch the extreme situations, which could cause the severe losses. The Scenario Analysis addresses the events that are statistically insignificant. In this project, some sensitivity tests are included to evaluate the extreme risks. The sensitivity tests include the change of financial risk factors and policyholder's behavior.

3. Evaluate Risk Management Strategies

The ultimate goal of risk management is not to eliminate risk completely or to hedge risk perfectly. An optimal risk management should be the most cost efficient way to reduce the risk below the company's risk tolerance, which then maximizes the company's risk-adjusted return. The alternative risk management techniques, evaluated below, are popular for hedging financial risks.

Put Options

The nature of the Rider guarantee is a series of embedded European put options with strike dates of ten years and more. The risks can be hedged very well by buying proper notional amount of put options on the right underlying indices. The hedging strategy is easy to implement; however, the put option with a strike date of ten years is not a standard option. A put option with a long expiration date can be purchased over-the-counter, known as the FLEX. The FLEX could be expensive because the option writers usually include a big margin. The fee collected from the policyholders might not be enough to cover the cost of buying the put options. It is also difficult to determine the right amount of the put option to purchase at the issue date due to policyholder behavior; for example, the policyholders can surrender the policy or partially withdraw the account value differently from the pricing assumptions. The risk associated with the policyholders' behavior is hard to hedge by a financial instrument. Using put options is very costly, and may not be an optimal risk management strategy.

Replicate or delta hedge

The insurance company acts like an option provider when it issues the Rider guarantee. The company can use the Black-Scholes equation to find a portfolio replicating the embedded put option. This hedge portfolio is established to mirror the change of the liabilities. This approach is gaining popularity in the actuarial profession due to its comprehensiveness and low-cost. But, it requires frequent rebalancing, which generates large transaction costs and significant hedging errors. The hedging might not hold in the extreme conditions, or when the Gamma is large. Moreover, the Black-Scholes assumptions, in practice, might not be realistic as well.

Life Insurance Securitization

Several insurance securitization deals have been done since 1996. The motivations for securitization are creating funding to sponsor insurance companies, to diversify funding sources, and to manage reserves associated with the new business strain and transferring insurance risks. With an increasing number of insurance securitizations, insurance securitization becomes an effective risk management method for some insurance companies. To manage the Rider guarantee, the company can securitize the additional reserves and required capital for the Rider. This approach can pass a big portion of the risk associated with the Rider guarantee to the investors. It also improves the rate of return for the products with the Rider significantly, if the internal required return is higher than the cost of the securitization.

The risks associated with life insurance securitization are more complex than a typical Asset-Backed security. The investors bear the primary insurance risks such as the mortality, lapse, and asset risks. They also take on the regulatory risk, service and operational risk. Life insurance securitization may not have a "true sale" like a typical Asset-Backed security has. Therefore, the investors in the securitization may be subject to bankruptcy of the insurance company. Because it is difficult for the investors to understand and quantify all these risks, they usually demand a higher return. The insurance company may also find life insurance securitization is a challenge, due to the lack of expertise and resources. Furthermore, securitization requires a qualified special purpose vehicle (SPV). Unfortunately, there are not many qualified SPVs currently. The cost associated with the modeling work, issuing and administration is significant; thus it is not worthwhile to securitize a small amount of reserve or required capital. The life insurance securitization is not an optimal choice for a small block of the GMAB business.

Semi-hedging with Equity Index Swap

Based on put-call parity, put options can be replicated by equity index swaps and call options. Instead of entering the swap agreement at the issue date, the company can enter the swap agreement when the account value goes down to a pre-set percentage. The pre-set percentage is determined based on the insurance company's risk tolerance. The insurance company pays the total return of the equity index and receives fixed income. When the market goes down further, the insurance company's loss is floored at the pre-set level. When the market goes up, the loss in the swap is offset by the small claims from the Rider guarantee. When the market goes up further, the insurance company would suffer an additional loss from the swap. Therefore, when the insurance company enters a swap, it also needs to buy a deep out-of-the-money call option on the index.

Using the equity index swap introduces a smaller hedge error than put options would generate. The company might not enter a swap agreement if the market does not go down to the pre-set percentage. This risk management strategy costs the company nothing in most cases. If the pre-set loss is small, the insurance company might need to enter a swap agreement with a greater chance. However, the insurance company might suffer a huge pre-set loss if the company has a greater risk tolerance. The pre-set tolerance loss level can be determined by optimization analysis, which requires a large amount of the work. A reasonable loss level was chosen to demonstrate this risk management process.

4. Evaluate the Risk Management System

The performance of a risk management system should be evaluated based on the extent that the overall objective was achieved. The actual loss amount can be used to back-test the VaR model, but it cannot evaluate the wellness of a risk management system. Once the objective of the risk management is defined, the risk manager is encouraged to minimize the costs of hedging, while trying to achieve the hedging objective within a certain confidence level. Hence, it promotes a cost efficient risk management strategy.

Risk management is a dynamic process, so risk management activities need to be monitored closely. The actual GMAB claim amounts can be used in the Back-testing to assess the accuracy of the VaR model, so it is necessary to keep an accurate history of them. The volatility of fund returns and the correlation between fund returns and indices should be re-calculated frequently, so the insurance company can set the Rider charge correctly and hedge the policies with an appropriate notional amount. The equity index swap rate and the total account value of each block of policies should be monitored closely as well, since it determines when the insurance company actually hedges a block of policies. To achieve risk optimization, the insurance company should revise or modify the risk management strategy when needed, and even change its risk management policy or objective accordingly.

Implement Risk Management Process

1. Implement Risk Measure

To show how to calculate the VaR and CTE at different percentiles, I built a stochastic model to carry out the Monte Carlo analysis. The Model includes 14 model points, representing different groups of annuity policies issued on July 1, 2006. All policies have the Rider elected with the GMAB amount that equals the premium paid. The charge of the Rider is 0.45% of the GMAB amount. The policies are grouped by age and size. Each policy is assumed to use the same investment allocation in 9 different mutual funds. The Model has $100 million dollar single-paid- premium in total.

The basic features of these policies are listed in the Appendix 1.
The major pricing model assumptions are listed in Appendix 2.

The mutual fund returns change over time, and vary by scenario. The total account value is based on the mutual fund performances. If the total account value is less than the GMAB amount (the guarantee is in-the-money) during the first ten years, The Model assumes no lapses. If the account value is less than the guaranteed surrender amount at the tenth year anniversary or thereafter, The Model assumes all policies surrender, and the insurance company pays the difference as the Rider benefit claim.

The GMAB amount can be reduced proportionally by partial withdrawals. The Model assumes a 2.5% penalty-free partial withdrawal rate per year. The Model also includes the reserve and C-3 Phase II required capital for the Rider. The Rider reserve and required capital affects the profitability, but has no impact on VAR and CTE measurements.

Calculate VaR and CTE

Using the Monte Carlo method to produce a distribution requires The Model to run numerous scenarios; in this project, a thousand scenarios are used. The estimation error would be approximately 3% (error of estimation equals 100% divided by the square root of the number of simulation, based on Hines and Montgomery 1980). Many variance reduction techniques are available today to reduce the standard error or the computation time. Since this project focuses on assessing the tail risk instead of evaluating the embedded option value, no variance reduction techniques will be employed.

For each scenario, The Model produces the Rider benefit claim for each policy if the account value is less than the guaranteed amount in the tenth anniversary or thereafter. In most cases, there are no benefit claims. Both benefit claims and fees collected are discounted back to the issue date for comparison. The loss from issuing the Rider is then calculated by subtracting the present value of fees collected from the benefit claims if it is greater than zero. Two discount rates are used to show the impact of interest rates on loss amounts and embedded option value.

To calculate the VaR, the loss amounts are ranked in ascending order, the smoothed empirical estimate of 95th percentile VaR is determined by linearly interpolating the loss amounts of the 950th and 951st scenarios. I also calculated the CTE 90 of loss amount by averaging the100 greatest loss amounts. The same calculation process is used to calculate the 95th percentile VaR and CTE 90 of benefit claim. The following chart includes the VaR and CTE for both loss amount and benefits claim using 5% interest rate to discount. A 5% discount rate is used as a representative risk-free rate for the 10-year treasury yield.

Table 1

	PV of Benefit Claim	PV of Loss Amount
95th Percentile VaR	$8,203,707	$5,250,632
97.5th Percentile VaR	$11,339,446	$8,318,246
CTE 65	$1,659,524	< 0
CTE 90	$5,808,332	$5,746,311

*Based on $100 Million Premium and 5% interest rate

If 6.5% interest rate is used to discount, both the loss amount and benefit claim decrease significantly. It shows that the value of the Rider moves inversely with interest rates.

Table 2

	PV of Benefit Claim	PV of Loss Amount
95th Percentile VaR	$7,118,805	$4,336,628
97.5th Percentile VaR	$9,839,856	$6,994,745
CTE 65	$1,383,996	< 0
CTE 90	$4,843,986	$4,764,914

*Based on $100 Million Premium and 6.5% interest rate

Assess Pricing Risk

The embedded option value is calculated using this Monte Carlo simulation to assess the pricing risk, because it is difficult to find a close form solution for the value of this option mentioned earlier. The value of the embedded option is the average benefit claim of the 1,000 scenarios, discounted back using the 10-year spot rate. It is a controversial issue whether the discount rate should include a spread reflecting the insurance company creditworthiness, or not. The 10-year treasury yield (5%) is used in this project. The embedded option value is 0.94% of the notional (GMAB) amount. Using the Black-

Scholes formula, the implied annualized volatility of 10 year total account returns is 10.4%. The historical data, from January 3, 1950 to February 6, 2006, of S&P 500 shows the annual volatility of S&P 500 index returns is approximately 15.7%. This implies the volatility of total account returns differs from the volatility of S&P 500. That is due to some separate account investment funds having less volatility than the S&P 500.

The present value of the annual fee income, which is currently 0.45% of guaranteed amount, is 3.16% of the GMAB amount. The fee income is sufficient to cover the loss with 90.9% probability. The maximum of the fee income, which is capped at 1% per year, would be 7.03% of the GMAB amount and sufficient to cover the loss with 93% probability. This is the fair value of the embedded option, although the volatility of the returns goes up to 19.6%. Obviously, the insurance company would suffer an implicit loss when the volatility of returns exceeds 19.6%.

The put option value has an inverse relationship with the discount rate. With a 6.5% discount rate, the embedded option value decreases from 0.94% to 0.81% of the GMAB amount. The annual fee income is sufficient to cover loss with 91.5% chance.

To include C3 Phase II RBC, the model includes CTE 65 as the reserve for the Rider and CTE 90 for the additional required capital for the Rider. The additional reserve and capital for the Rider will not affect the VaR and CTE amount, but it has a big impact on profitability. The following chart shows the impact of including the Rider on a variable annuity.

Table 3

	Without IPP Rider		With IPP Rider	
	Rate of Return	Embedded Value	Rate of Return	Embedded Value
Mean	15.4%	$2,476,290	13.3%	$3,045,300
90th percentile	5.8%	-$620,139	5.7%	-$1,258,094
Standard deviation		$2,809,472		$3,952,922

*Embedded Value is calculated by discounting distributable income at assumed 8% Weighted Average Cost of Capital

The comparison indicates the Rider generates an additional value to the company, but it requires more capital and risk tolerance. However, a proper risk management strategy can lessen the Rider risk.

Test Sensitivity

As mentioned previously, sensitivity tests can be used to test extreme risks. By assuming the premiums are allocated into three more volatile funds—25% in international equity fund, 25% in small cap mutual fund and 50% aggressive equity fund, both VaR and CTE increase significantly. The embedded option value increases from 0.94% to 1.83% of the GMAB amount. The fee income is now sufficient to cover the loss with 85.8% probability instead of 90.9% probability. The embedded option value can be even higher in a low interest rate environment. It shows that the insurance company should increase the rider charge when the volatility of the fund returns increases. The table below shows the VaR and CTE amount if policyholders switch to more volatile funds.

Table 4

	PV of Benefit Claim	PV of Loss Amount
95th Percentile VaR	$13,039,194	$9,986,312
97.5th Percentile VaR	$19,533,304	$16,555,633
CTE 65	$3,620,023	$2,223,668
CTE 90	$11,912,749	$11,912,749

*Based on $100 Million Premium and 5% interest rate

Policyholders have a right to cancel the Rider, but they will get charged a fee of 2% of guaranteed amount fee. If the account value is much higher than the GMAB, the Rider will become worthless. Policyholders may not be willing to pay the 0.45% fee when the account value is high and GMAB is out-of-the-money. They are more likely to surrender their policy after the surrender charge period instead of paying the 2% Rider cancellation fee. Therefore, the fee income from the Rider will be smaller than what it would be under a normal lapse assumption. The profitability will be worse also. The comparison between the normal lapse and additional surrender assumption (as described on appendix 2) is in the following chart.

Table 5

	Normal Lapse Assumption	Extra Surrender Assumption
Mean IRR	13.32%	10.36%
Averaged Embedded Value	$2,971,896	$933,251
Averaged PV of Benefit Claim	$936,469	$901,018
Averaged PV of Fee Income	$3,163,651	$2,579,820

*Based on $100 Million Premium and 5% interest rate

A sensitivity test was also done for sales over a time period of five years in the future to see how multiple years of sales would interact, since each year of sales would have different times for having a rider benefit being in-the-money. The results show the profitability and VaR only vary slightly for the sales in different years. As long as the charges of the Rider correctly reflect the return volatility, and the interest rate level, and the timing of policyholder election the Rider will not increase risk exposure, nor hurt the profitability.

2. Implement Risk Management Strategy

Set Risk Tolerance Level

The fee income from the Rider charge is 0.45% of the GMAB annually. The accumulated value of the fee income would be 5.78% of GMAB at the tenth year, assuming 5% accumulation rate. Let's assume that the insurance company is willing to accept the risk of losing the fee income.

To determine when the insurance company should hedge this policy, the fixed income received by entering the equity index swap needs to be taken into account. Assuming the swap exchanges 5% YTM bond to an equity index, a pre-set tolerance loss level can be determined by the amount of fixed income return plus 5.78% of the GMAB amount. The notional amount will be the account value at the time of entering the swap. For example, the insurance company enters a four-year equity index swap with a notional amount of $7,752 at the end of the sixth policy year for a policy with $10,000 GMAB. The company will receive $9,422 ($7,752 accumulated at 5% for four years) from the swap agreement at the end of the tenth policy year. Of course, the company will pay the total return of the equity index applied to $7,752. With $578 the accumulated value fee income from the Rider charge (5.78% of the GMAB amount), the company will have $10,000 ($9,422 + $578) to pay the GMAB claim at the end of the tenth policy year. Therefore, the pre-set tolerance loss level for a policy with $10,000 GMAB at the end of the sixth policy year is $2,248 ($10,000 - $7,752). The pre-set tolerance loss level will vary by policy years since the company needs to enter different equity index swaps with different maturities. The pre-set tolerance account value for entering the swap is the guaranteed amount less the tolerance loss level. The following chart shows the tolerance account value at the end of each year (as a percentage of GMAB amount) for entering swap agreement.

Table 6

Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
63.8%	67.0%	70.3%	73.8%	77.5%	81.4%	85.5%	89.7%

Use Equity Index Swap and Call Option

Here is an example to illustrate using an equity index swap. The insurance company hedges a policy with $60,000 GMAB by entering an equity-index swap when the account value drops to $46,509 (77.5% of $60,000) at the end of the sixth year. The swap exchanges the total return of the equity index for the total return of 5% YTM bond four years later. The insurance company also purchases a deep out-of-the-money European call on the equity index at a small cost. The call with the strike price of $60,000 will expire 4 years later. To preserve the risk tolerance, the tolerance account value should be adjusted by the cost of the call option. In this project, the hedging is carried out without purchasing call options.

The least variance hedge technique should be applied in calculating the notional amount. To simplify the illustration, assume the account value has a perfect correlation with index movement. That is, one dollar of gain in the account value will occur at the same time that the index also gains one dollar using the calculated notional amount.

The account value can be less than $46,509, between $46,509 and $60,000, or greater than $60,000 at the end of the tenth year. The insurance company would lose a pre-determined amount regardless of what the account value is at the end of the tenth year. The table below shows that the loss amounts of the three scenarios are all within the range of the risk tolerance at the end of the tenth year.

Table 7

Account Value	Index Level	Payment to the swap counterparty	Receive from the swap counterparty	GMAB Claim	Call Option payoff	Total Loss
$40,000	$40,000	$40,000	$56,532	$20,000	$0	$3,468
$58,000	$58,000	$58,000	$56,532	$2,000	$0	$3,468
$65,000	$65,000	$65,000	$56,532	$0	$5,000	$3,468

The $56,532 received from the swap counterparty is notional amount $46,509 accumulated at 5% for four years.

Assess the Hedging

Assume that a person buys a policy with $60,000. They hold onto the policy for more than 10 years and have never taken a partial withdrawal. The guarantee amount of the Rider is $60,000. The policyholder invests 60% in equity-like funds and 40% in the fixed account. Based on the tolerance account value, the probability for the insurance company to hedge this policy varies by policy duration as in the following chart. It shows the insurance company doesn't need to hedge in most cases. When hedging is needed, the cost of hedging is minimized. The cost mainly is the deep out-of-the-money call plus the possible hedging error.

Table 8

Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
0.3%	0.5%	1.3%	2.6%	4.1%	5.5%	6.6%	8.6%

The insurance company is assumed not to hedge the loss within the pre-set tolerance loss level. If the insurance company implements this hedging strategy starting from the fifth policy year, it would experience un-hedged losses with a 5.4% chance. The average un-hedged loss is 6.98% of the GMAB. Lowering the tolerance loss level increases the chance the insurance company actually hedges the policy. On the other hand, it decreases the probability that the insurance company will experience the un-hedged loss. This hedging strategy reduces the VaR and CTE significantly, and hedges the tail risk well. The table shows the VaR without any hedging and VaR using the equity index swap hedging (call options are not included).

Table 9

	Loss without hedge	Loss with hedge
95th Percentile VaR	$10,929	$4,940
99th Percentile VaR	$16,723	$9,667
CTE 80	$6,137	$3,717
CTE 90	$11,118	$5,883

*Based on $60,00 Premium and 5% interest rate

The cost efficiency is the major advantage of this hedging strategy. The insurance company can enter a swap agreement without an upfront premium, and purchase a deep out-of-the-money call, probably with little cost. If the insurance company is willing to take an additional risk, this hedging strategy can be carried out without purchasing the call option. The additional risk will be a loss in the swap contract when the equity market goes up rapidly. Without purchasing the call option, there is a small chance the insurance company has a small loss amount in the swap, as shown in the following table.

Table 10

	Year 5	Year 6	Year 7	Year 8	Year 9
Probability of loss in swap Based on all scenarios	0.1%	0.4%	0.6%	0.3%	0.1%
Probability of loss in swap Based on only those scenarios being hedged	3.8%	9.8%	10.9%	4.5%	1.2%
Average loss amount of swap for hedged scenarios	4.9%	7.6%	6.4%	6.8%	7.3%

The risk deriving from the dynamic policyholder (lapse) behavior can be minimized by using this hedging strategy, because the policyholders are unlikely to surrender their policy once the guarantee is in-the-money. The mortality can be estimated fairly well with a large number of policies. Therefore, the amount to be hedged can be determined fairly accurately, since there is not expected to be much variance from the pricing assumptions.

Remaining Risks

It is almost impossible to eliminate the Rider risks completely. There are still significant risks remaining. There can be large base risk if we are hedging multiple fund performances with one or two indices. It can be reduced by using more indices, including equity indices and fixed income indices. Setting a tolerance loss level, the insurance company inherits the risk of losing the fee income, and with small probabilities may loss even more. Uncertainty of the call option price can be another risk to implementing this hedging strategy. The insurance company may want to enter a swap agreement after the fifth policy year, so a FLEX call option can be avoided. When the account value is close to the guarantee amount during the tenth year, the insurance company may use a put option instead of this hedging strategy.